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# Effects of the Space-like Penguin Diagrams on $CP$ Asymmetries in Exclusive $B$ Decays

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## Abstract

The space-like penguin contributions were ignored on little ground in many previous studies of nonleptonic  $B$  transitions and  $CP$  violation. Taking the penguin-dominated channels  $B_u^- \rightarrow \bar{K}^0 \pi^-$  and  $B_u^- \rightarrow K^0 K^-$  for example, we illustrate the non-negligible effects of the space-like penguin diagrams on  $CP$  asymmetries. Some qualitative remarks are given on the gluonic penguin picture and direct  $CP$  violation in exclusive two-body decays of  $B$  mesons.

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Within the standard electroweak model, direct  $CP$  violation is expected to manifest itself significantly in some exclusive  $B$ -meson decays which are dominated by the one-loop gluonic penguin transitions  $b \rightarrow qg^* \rightarrow q(q_g\bar{q}_g)$  ( $q = d$  or  $s$  and  $q_g = u, d$ , or  $s$ ) [1-4]. Non-zero  $CP$  asymmetries can arise through the interference between two independent amplitudes that have both different  $CP$  violating phases and different  $CP$  conserving phases. In the time-like penguin diagram, the necessary strong phases are provided by different loop effects of internal  $u$  and  $c$  quarks involving real (on-shell) particle rescattering. In contrast, the space-like penguin diagram can only provide an overall  $CP$  conserving phase due to final state hadronization. Thus the time-like penguin amplitudes play the key role in giving rise to  $CP$  violation, while the space-like penguin amplitudes only take effects by modifying the dispersive or absorptive parts of the time-like ones. In many previous studies of  $CP$  violation in  $B$  decays, the contributions from those annihilation-type diagrams were neglected for the argument that they should be formfactor suppressed or helicity unfavoured. This argument becomes questionable, however, for the space-like penguin channels since their amplitudes can be remarkably enhanced by the hadronic matrix elements involving  $(V - A)(V + A)$  or  $(S - P)(S + P)$  currents [5]. To date, a phenomenological demonstration of the space-like penguin effects on  $CP$  asymmetries in exclusive  $B$  decays has been lacking.

In this work we shall take the penguin-dominated decay modes  $B_u^- \rightarrow \bar{K}^0\pi^-$  and  $K^0K^-$  for example to illustrate the non-negligible contributions of the space-like penguin amplitudes to  $CP$  asymmetries. A simple kinematic picture is presented for decays of the type  $B \rightarrow PP$  in order to minimize the uncertainty with the gluonic momentum transfer. We calculate decay amplitudes by using the effective weak Hamiltonian and factorization approximation. Our numerical results show that the space-like penguin diagrams can significantly affect the  $CP$  violating signals in exclusive  $B$  transitions. It is therefore worthwhile to reexamine some previous works on the penguin-dominated  $B$  decays and  $CP$  violation, in which the space-like penguin effects were ignored on little ground.

The transitions  $B_u^- \rightarrow \bar{K}^0\pi^-$  and  $B_u^- \rightarrow K^0K^-$  are of great interest for studying direct  $CP$  violation [1-4] and extracting the Kobayashi-Maskawa (KM) phase parameters [6]. They have two advantages for our present purpose. First, they occur only through the tree-level annihilation and penguin channels in the quark-diagram scheme [5,7]. The former can be safely neglected in comparison with the space-like penguin contribution. Second, the electroweak penguin effects on these two decay modes are rather small [8], thus one may simply apply the

QCD-loop induced Hamiltonian to them. The one-loop gluonic penguin Hamiltonian for  $B$  decays is given by [1-5]

$$\mathcal{H}_{\text{eff}}(\Delta B = -1) = -\frac{G_F}{\sqrt{2}} \frac{\alpha_s}{8\pi} \left[ \sum_i v_i (F_i^{\text{T}} + F_i^{\text{S}}) \right] \left( -\frac{Q_3}{N_c} + Q_4 - \frac{Q_5}{N_c} + Q_6 \right), \quad (1)$$

where QCD corrections are approximately included by the effective coupling constant  $\alpha_s$  at the physical scale  $\mu = m_b$ ;  $v_i = V_{ib}V_{iq}^*$  ( $i = u, c, t$  and  $q = d, s$ ) are the KM factors corresponding to  $b \rightarrow q$ ;  $F_i^{\text{T}}$  and  $F_i^{\text{S}}$  stand for the loop integral functions of the time-like (T) and space-like (S) penguin diagrams respectively;  $N_c$  is the number of colors; and  $Q_{3,\dots,6}$  represent gluonic penguin operators of the form [9]

$$\begin{aligned} Q_3 &= (\bar{q}b)_{V-A} \sum_{q_g} (\bar{q}_g q_g)_{V-A}, & Q_4 &= (\bar{q}^\alpha b^\beta)_{V-A} \sum_{q_g} (\bar{q}_g^\beta q_g^\alpha)_{V-A}, \\ Q_5 &= (\bar{q}b)_{V-A} \sum_{q_g} (\bar{q}_g q_g)_{V+A}, & Q_6 &= (\bar{q}^\alpha b^\beta)_{V-A} \sum_{q_g} (\bar{q}_g^\beta q_g^\alpha)_{V+A}. \end{aligned} \quad (2)$$

Note that  $F_i^{\text{T,S}}$  are dependent on the gluonic momentum transfer  $k^2$ . Without loss of generality, we obtain the analytical expressions of  $F_i^{\text{T}}$  and  $F_i^{\text{S}}$  as [10]

$$\begin{aligned} F_{i=u,c}^{\text{S}} &= \frac{10}{9} - \frac{2}{3} \ln a_i + \frac{2}{3} r_i - \frac{1}{3} (2 + r_i) \sqrt{1 - r_i} \left[ \ln |1 + \sqrt{1 - r_i}| - \ln |1 - \sqrt{1 - r_i}| \right], \\ F_{i=u,c}^{\text{T}} &= \frac{10}{9} - \frac{2}{3} \ln a_i + \frac{2}{3} r_i - \frac{1}{3} (2 + r_i) \sqrt{|1 - r_i|} \left[ 2 \operatorname{arccot} \sqrt{r_i - 1} \theta(r_i - 1) \right. \\ &\quad \left. + \left( \ln |1 + \sqrt{1 - r_i}| - \ln |1 - \sqrt{1 - r_i}| - i\pi \right) \theta(1 - r_i) \right], \\ F_{i=t}^{\text{S}} &= F_{i=t}^{\text{T}} = \frac{(18 - 11a_i - a_i^2)a_i}{12(1 - a_i)^3} - \frac{4 - 16a_i + 9a_i^2}{6(1 - a_i)^4} \ln a_i, \end{aligned} \quad (3)$$

where  $a_i = m_i^2/m_W^2$ ,  $r_i = 4m_i^2/k^2$ , and  $F_{i=t}^{\text{S}} = F_{i=t}^{\text{T}} = 43/72$  at the limit  $m_t = m_W$ .

Between a decay mode  $B_u^- \rightarrow f$  and its  $CP$ -conjugate counterpart  $B_u^+ \rightarrow \bar{f}$ , the  $CP$  asymmetry  $\mathcal{A}_f$  is defined as the ratio of the difference to the sum of their decay rates. Since  $v_u + v_c + v_t = 0$ , each decay amplitude can be decomposed into two terms which are proportional to  $v_u$  and  $v_c$  respectively. Hence  $\mathcal{A}_f$  depends only upon the KM phase  $(v_u v_c^*)$  and the quantity

$$R_f = \frac{(F_c^{\text{T}} - F_t^{\text{T}}) + \xi_f (F_c^{\text{S}} - F_t^{\text{S}})}{(F_u^{\text{T}} - F_t^{\text{T}}) + \xi_f (F_u^{\text{S}} - F_t^{\text{S}})}, \quad (4)$$

consisting of the non-trivial strong phases due to final state interactions. Here the parameter  $\xi_f$  measures the relative size and sign between the space-like and time-like penguin amplitudes. In many previous studies  $\xi_f = 0$  was assumed. Explicitly the  $CP$  asymmetry  $\mathcal{A}_f$  can be expressed as

$$\mathcal{A}_f = \frac{2\operatorname{Im}(v_u v_c^*) \cdot \operatorname{Im} R_f}{|v_u|^2 + |v_c|^2 \cdot |R_f|^2 + 2\operatorname{Re}(v_u v_c^*) \cdot \operatorname{Re} R_f}. \quad (5)$$

In the following we shall discuss how to evaluate the loop intergral functions  $F_i^{\text{T,S}}$  and the hadronization parameter  $\xi_f$  in order to quantitatively determine  $R_f$  and  $\mathcal{A}_f$ .

The problem with  $F_i^{\text{T}}$  and  $F_i^{\text{S}}$  is the unknown value of  $k^2$ , the four-momentum squared of the virtual gluon in the exclusive penguin channels. For the time-like penguin transitions, one used to pick a special value from  $k^2 \in (0, m_b^2]$  or  $k^2 \in [m_b^2/4, m_b^2/2]$  with some kinematic arguments [4]<sup>2</sup>. Taking the space-like penguin diagrams into account, here we present a simple kinematic picture for two-body penguin-induced decays  $B \rightarrow XY$  as illustrated in Fig. 1. In the rest frame of the  $B$  meson, we assume: (a) the spectator quark of the time-like penguin graph has negligibly small momentum in either the initial or the final state; (b) the two quarks forming the meson  $X$  in Fig. 1(a) have the same momentum; and (c) the momentum of the quark pair created from the vacuum are negligible in the space-like penguin graph Fig. 1(b). Accordingly we find that the average value of the gluonic momentum transfer  $k^2$  can be given by

$$\langle k^2 \rangle = m_b^2 + m_q^2 - 2m_b E_q, \quad (6)$$

where  $E_q$  is determinable from

$$E_q + \sqrt{E_q^2 - m_q^2 + m_{q_g}^2} + \sqrt{4E_q^2 - 4m_q^2 + m_{q_g}^2} = m_b \quad (7a)$$

for the time-like penguin channels; or from

$$E_q + \sqrt{E_q^2 - m_q^2 + m_{q_g}^2} = m_b + m_{q_g} \quad (7b)$$

for the space-like penguin channels. In the case of  $q = q_g$ , we obtain  $\langle k^2 \rangle_{\text{T}} = \frac{1}{2}(m_b^2 - m_q^2) > 0$  and  $\langle k^2 \rangle_{\text{S}} = m_q(m_q - m_b) < 0$ . From Eq. (7) we observe that taking  $|\langle k^2 \rangle_{\text{T}}| = |\langle k^2 \rangle_{\text{S}}|$  is in general not true for phenomenology.

The above kinematic picture, based on the valence-quark assumption, has avoided the drawback of taking arbitrary values for  $k^2$  in the studies of exclusive penguin-mediated  $B$  decays. However, such an analysis cannot reflect the dynamics of final state hadronization. For example, it predicts the same size of  $\langle k^2 \rangle_{\text{T}}$  (or  $\langle k^2 \rangle_{\text{S}}$ ) for the processes  $B_u^- \rightarrow K^{(*)-} + (\pi^0, \eta^0, \rho^0)$ . Hence it is problematic to apply this approach to all two-body  $B$  transitions. Pursuing a

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<sup>2</sup>Considering an additional hard gluon to accelerate the spectator quark in a time-like penguin graph, Simma and Wyler [1] have calculated the  $k^2$  distribution of some charmless exclusive  $B$  decays and folded it with the momentum dependence of the loop amplitudes. The relevant branching ratios yielded in this method are however smaller than those from other ways.

phenomenological insight into the time-like and space-like penguin diagrams, we believe that the simple picture given above should be applicable to those  $B$  decays into two pseudoscalar mesons. Whether parallel estimates of  $\langle k^2 \rangle$  can be carried out for penguin-induced  $B$  decays of the types  $B \rightarrow PV$  and  $B \rightarrow VV$  is still an open question.

Applying the effective Hamiltonian  $\mathcal{H}_{\text{eff}}$  and factorization approximation [11] to  $B_u^- \rightarrow \bar{K}^0 \pi^-$  and  $K^0 K^-$ , one obtains

$$\begin{aligned}\xi_{\bar{K}^0 \pi^-} &= \left[ 1 + \frac{2m_{B_u^-}^2}{(m_s - m_u)(m_b + m_u)} \right] \cdot \left[ 1 + \frac{2m_{\bar{K}^0}^2}{(m_s + m_d)(m_b - m_d)} \right]^{-1} \cdot Z_{\bar{K}^0 \pi^-}, \\ \xi_{K^0 K^-} &= \left[ 1 + \frac{2m_{B_u^-}^2}{(m_d - m_u)(m_b + m_u)} \right] \cdot \left[ 1 + \frac{2m_{K^0}^2}{(m_d + m_s)(m_b - m_s)} \right]^{-1} \cdot Z_{K^0 K^-}.\end{aligned}\quad (8)$$

In this equation, the terms proportional to  $m_{B_u^-}^2$  ( $m_{\bar{K}^0}^2$  or  $m_{K^0}^2$ ) arise from transforming the  $(V - A)(V + A)$  currents into the  $(V - A)(V - A)$  ones for the space-like (time-like) penguin amplitudes; and the parameter  $Z_f$  ( $f = \bar{K}^0 \pi^-$  or  $K^0 K^-$ ) describes the ratio of the space-like hadronic matrix element to the time-like one after factorization. In terms of decay constants and formfactors [12,5],  $Z_{\bar{K}^0 \pi^-}$  and  $Z_{K^0 K^-}$  can be expressed as

$$\begin{aligned}Z_{\bar{K}^0 \pi^-} &= -\frac{m_{\bar{K}^0} - m_{\pi^-}}{m_{\bar{K}^0} + m_{\pi^-}} \cdot \frac{m_{B_u^-} + m_{\pi^-}}{m_{B_u^-} - m_{\pi^-}} \cdot \frac{(m_{\bar{K}^0} + m_{\pi^-})^2 - m_{B_u^-}^2}{(m_{B_u^-} + m_{\pi^-})^2 - m_{\bar{K}^0}^2} \cdot \frac{f_{B_u^-} F_+^a(m_{B_u^-}^2)}{f_{\bar{K}^0} F_+^{B_u^- \pi^-}(m_{\bar{K}^0}^2)}, \\ Z_{K^0 K^-} &= -\frac{m_{K^0} - m_{K^-}}{m_{K^0} + m_{K^-}} \cdot \frac{m_{B_u^-} + m_{K^-}}{m_{B_u^-} - m_{K^-}} \cdot \frac{(m_{K^0} + m_{K^-})^2 - m_{B_u^-}^2}{(m_{B_u^-} + m_{K^-})^2 - m_{K^0}^2} \cdot \frac{f_{B_u^-} F_+^a(m_{B_u^-}^2)}{f_{K^0} F_+^{B_u^- K^-}(m_{K^0}^2)},\end{aligned}\quad (9)$$

where the annihilation formfactor  $F_+^a(m_{B_u^-}^2) = i16\pi\alpha_s f_{B_u^-}^2/m_{B_u^-}^2$  is given by QCD calculations [13]. It should be noted that  $Z_f$  or  $\xi_f$  is primarily absorptive and is not negligible. The presence of the space-like penguin amplitudes can correct  $\text{Im}F_i^T$  through Eq. (4). In contrast, the perturbative QCD corrections mainly modify  $\text{Re}F_i^T$ , as shown in Ref. [3]. Thus evaluating the size of  $Z_f$  is very desirable in order to properly calculate  $CP$  asymmetries in the penguin-mediated  $B$  decays.

For illustration, let us estimate the  $CP$  asymmetry  $\mathcal{A}_f$  ( $f = \bar{K}^0 \pi^-$  or  $K^0 K^-$ ) and compare the result of  $\xi_f \neq 0$  with that of  $\xi_f = 0$ . We use the current quark masses  $(m_u, m_d, m_s, m_c, m_b, m_t) = (0.005, 0.01, 0.175, 1.35, 4.8, 170)$  in unit of GeV and the meson masses  $(m_{\pi^-}, m_{K^0}, m_{K^-}, m_{B_u^-}) = (139.6, 497.7, 493.7, 5278.7)$  in unit of MeV [14]. The decay constants and formfactors are taken as  $f_{\pi^-} = 130.7$  MeV,  $f_{K^0} = f_{\bar{K}^0} \approx f_{K^-} = 159.8$  MeV [14],  $f_{B_u^-} \approx 1.5f_{\pi^-}$  [15];  $F_+^{B_u^- \pi^-}(0) \approx 0.29$ , and  $F_+^{B_u^- K^-}(0) \approx 0.32$  [16]. The inputs of the Wolfenstein parameters [17] are  $A = 0.80$ ,  $\lambda = 0.22$ ,  $\rho = -0.07$ , and  $\eta = 0.38$  [18]. After a straightforward calculation we find  $\xi_{\bar{K}^0 \pi^-} \approx i1.3$ ,  $\mathcal{A}_{\bar{K}^0 \pi^-} \approx 1.3\%$  and  $\xi_{K^0 K^-} \approx i0.32$ ,

$\mathcal{A}_{K^0 K^-} \approx -10.9\%$ . In comparison with the values  $\mathcal{A}_{\bar{K}^0 \pi^-} \approx 0.4\%$  and  $\mathcal{A}_{K^0 K^-} \approx -6.4\%$ , which are obtained by taking  $\xi_{\bar{K}^0 \pi^-} = \xi_{K^0 K^-} = 0$ , we observe that contributions from the space-like penguin amplitudes can significantly enhance the  $CP$  asymmetries.

More generally one may assume an arbitrary phase shift  $\theta_f$  between the space-like and time-like penguin amplitudes. Replacing  $\xi_f$  by  $|\xi_f|e^{i\theta_f}$  in Eq. (4), we examine the dependence of  $\mathcal{A}_f$  upon  $\theta_f$  for the decay modes  $B_u^- \rightarrow \bar{K}^0 \pi^-$  and  $K^0 K^-$ . The numerical results are shown in Fig. 2, where  $\theta_{\bar{K}^0 \pi^-}$  and  $\theta_{K^0 K^-}$  change from  $-180^\circ$  to  $180^\circ$ . It is clear that the space-like penguin transitions play an important role for  $CP$  violation in these two processes.

Certainly the numbers given above have many uncertainties that are unable to be removed to the limit of our present understanding of nonleptonic weak decays and nonperturbative confinement forces [19]. While the quantitative results might not be trustworthy, we emphasize that qualitatively effects of the space-like penguin diagrams on  $CP$  asymmetries (and branching ratios) in exclusive  $B$  decays should be non-negligible. Whether such effects are constructive or destructive to  $CP$  violating signals depends upon final state interactions of the decay modes under discussion. Our conclusion is that further studies of the penguin-dominated  $B$  decays (in particular, those promising processes for probing direct  $CP$  violation or testing unitarity of the KM matrix) are very necessary in order to advance our understanding of the “penguin physics” [20] and the underlying mechanism of direct  $CP$  violation.

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## Figure Captions

**Figure 1:** Quark diagrams for a  $B$  meson decaying into two light mesons  $X$  and  $Y$  through the gluonic penguin process  $b \rightarrow qg^* \rightarrow q(q_g\bar{q}_g)$ : (a) the time-like penguin, and (b) the space-like penguin. The dark dot stands for the effective four-fermion interactions  $Q_3 - Q_6$ . The subscripts “s” and “v” denote “spectator” and “vacuum”, respectively.

**Figure 2:** Dependence of the  $CP$  asymmetry  $\mathcal{A}_f$  upon the phase shift  $\theta_f$  between the space-like and time-like penguin amplitudes: (a)  $B_u^- \rightarrow \bar{K}^0\pi^-$ , and (b)  $B_u^- \rightarrow K^0K^-$ .

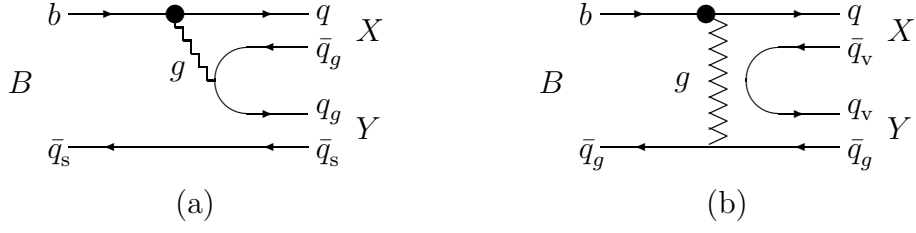


Figure 1:

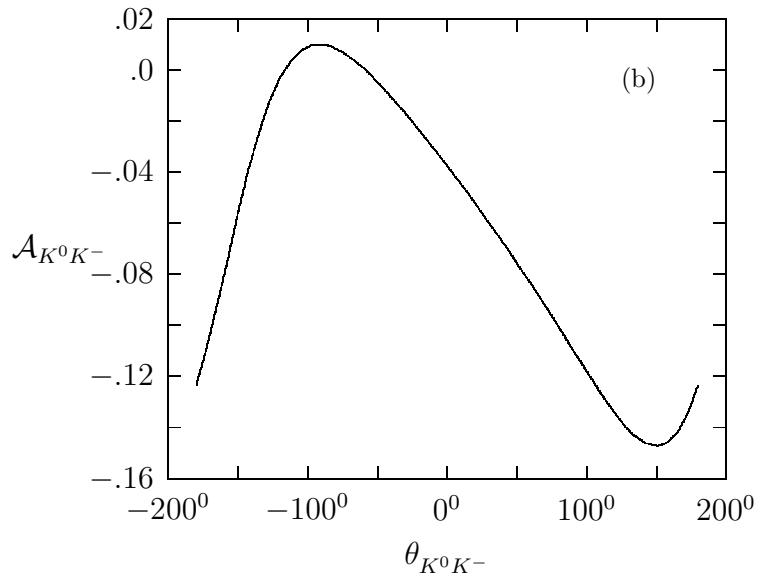
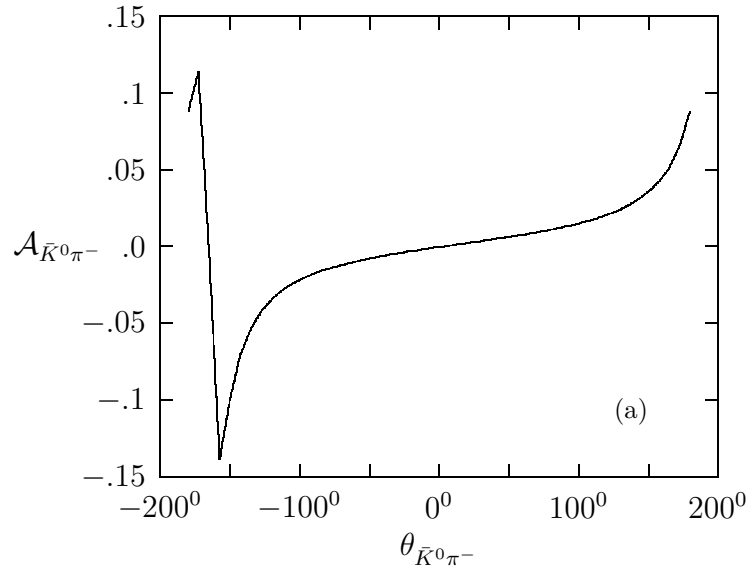


Figure 2: